Intro to Probability
Day 3
(Compound events & their probabilities)

Compound Events

Let A, and B be two event. Then we can define 3 new events as follows:

1) A *or* B (also $A \cup B$)
…is the list of all outcomes in A together with those in B
(i.e. an outcome is in A or B if it's in A or in B or both)

2) A *and* B (also $A \cap B$)
…is the list of all outcomes that are in both A and B

3) *not* A (also $\bar{A}$)
…is the list of all outcomes of S that are not in A

Definition: Events A and B are <u>disjoint</u> or <u>mutually exclusive</u> if $A \cap B = \emptyset$.

Formulas for $P(A \cup B)$, $P(A \cap B)$, & $P(\bar{A})$

Formulas for $P(A \cup B)$ (the addition rule)

1) If A and B are disjoint (i.e. $A \cap B = \emptyset$), then

$$P(A \cup B) = P(A) + P(B)$$

2) If A and B are NOT disjoint (i.e. $A \cap B \neq \emptyset$), then

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Ex 1:

Experiment

Play a single game of roulette

Events

A = The ball lands in an odd slot

B = The ball lands in a red slot

C = The ball lands in a green slot

D = The ball lands on a number that is a multiple of 3

E = The ball lands in a slot that is part of the 1st 12 bet

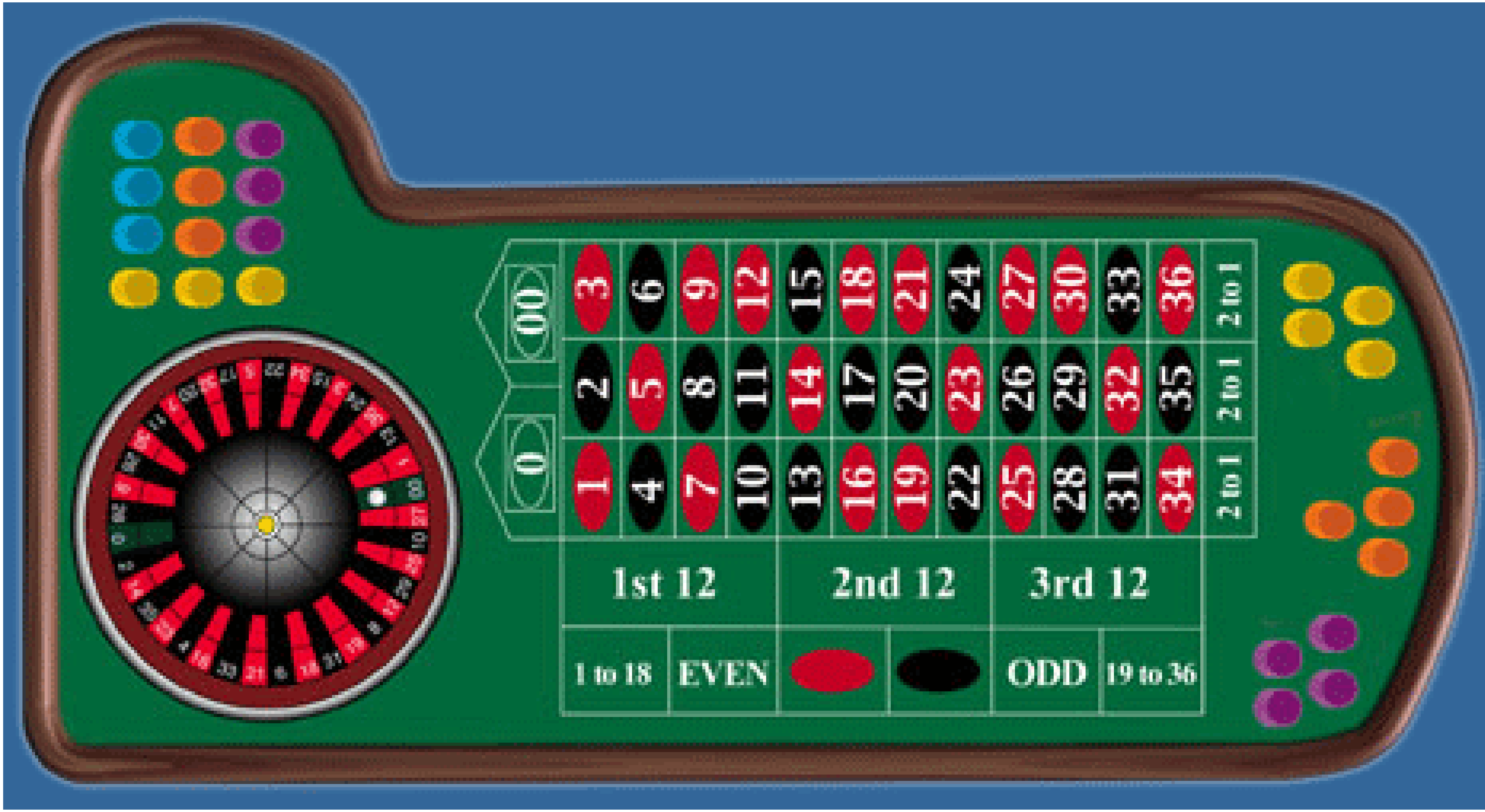
F = The ball lands in a slot that is part of the 3rd 12 bet and is a black number

G = The ball lands on a number that is part of the 2nd column

Find

$P(A \cup B)$, $P(C \cup E)$

Ex 1: Picture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0
3 6 9 12 15 18 21 24 27 30 33 36
2 5 8 11 14 17 20 23 26 29 32 35
1 4 7 10 13 16 19 22 25 28 31 34
2 to 1
2 to 1
2 to 1
1st 12
2nd 12
3rd 12
1 to 18
EVEN
ODD
19 to 36

Ex 2: A blood bank catalogs the types of blood, including positive or negative Rh-factor, given by donors during the last five days. The number of donors who gave each blood type is shown in the table. A donor is selected at random.

a) Find the probability that the donor has type O or type A blood.

b) Find the probability that the donor has type B blood or is Rh-negative.

		Blood Type				
		O	A	B	AB	Total
Rh-factor	Positive	156	139	37	12	344
	Negative	28	25	8	4	65
	Total	184	164	45	16	409

Ex 3: The probability that a randomly selected person in the world will have brown eyes is 53%, the probability they will have blue eyes is 18%, the probability they will have green eyes is 16%, the probability they will be left handed is 10% and the probability that they will have blue eyes and be left handed is 2%.

a) What is the probability that a randomly selected person in the world will have brown eyes or green eyes?
b) What is the probability that a randomly selected person in the world will have blue eyes or be left handed?
c) What is the probability that a randomly selected person in the world will have brown eyes, blue eyes, or green eyes?

Conditional Probability

Let A, and B be two events. Then $P(A|B)$ (read probability of A given B) is the probability of the event A given that the event B has already occurred.

Notes:

1) When finding $P(A|B)$, A is the event that you are finding the probability of given the new information that the event B has occurred.
2) When calculating $P(A|B)$, THE SAMPLE SPACE CHANGES. B helps you figure out what the new sample space is.
3) $P(A|B)$ is usually not the same as $P(B|A)$

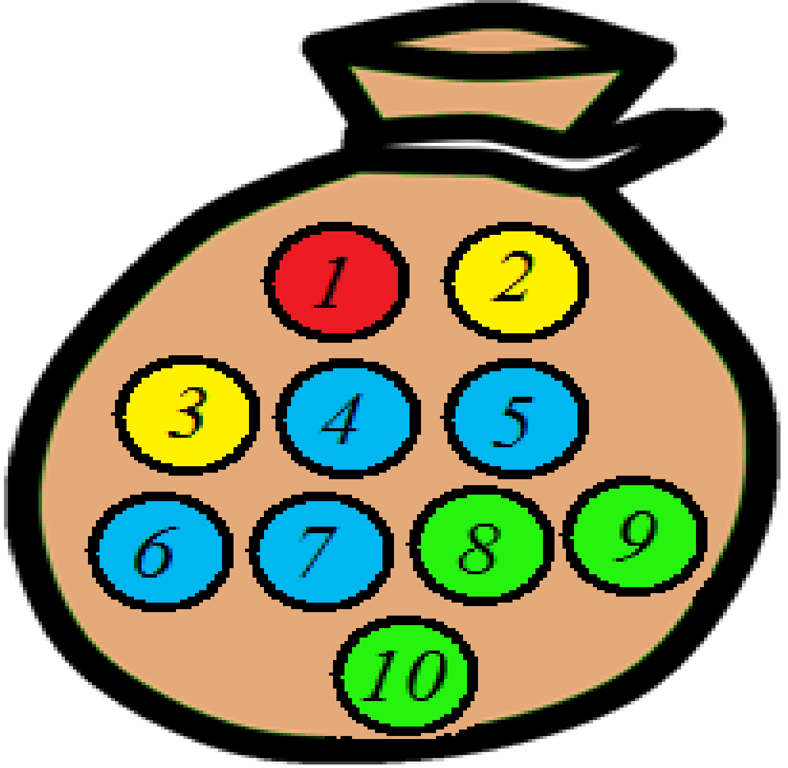
Definition: Events A and B are <u>independent</u> if $P(A|B) = P(A)$

Note: When doing problems with multiple draws, if the draws are done with replacement then each draw is independent of the others, if the draws are done without replacement then the each draw depends on the others.

Ex 4:

Experiment

Draw a single ball from the bag →



Events

A = You draw a ball with an even number on it

B = You draw a ball with a prime number on it

C = You draw a yellow ball

D = You draw a ball that is both blue and even

E = You draw a ball with a number larger than 6 on it

F = You draw a ball that is both less than 7 and odd

Find

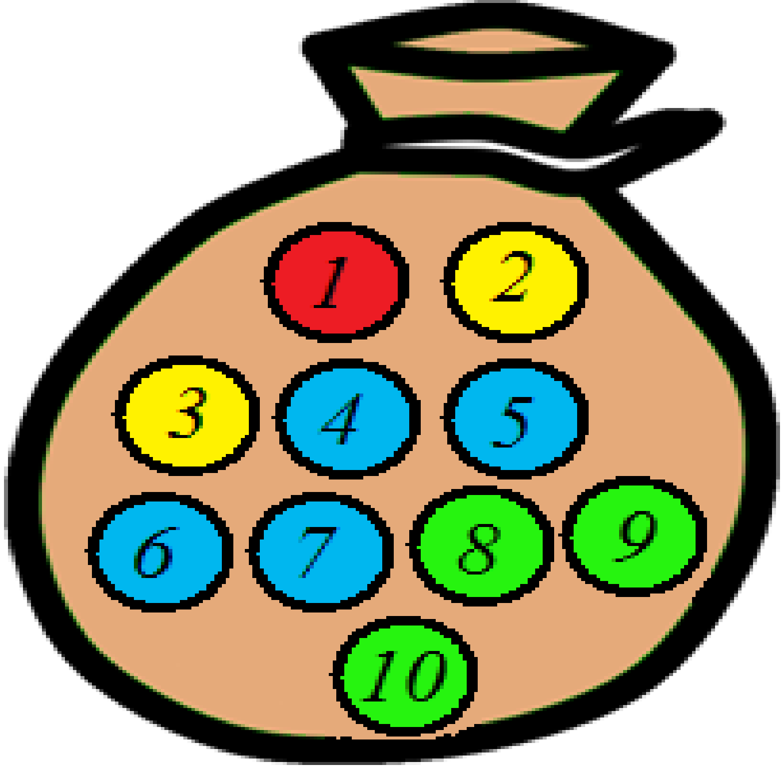
$P(A|B)$, $P(B|A)$, $P(C|F)$, $P(B|E)$

Question: Are the events *A* and *C* independent? How about the events *B* and *E*?

Ex 5:

Experiment

Draw 2 balls from the bag one by one with replacement



Events

R_i = The ith ball drawn is red

Y_i = The ith ball drawn is yellow

B_i = The ith ball drawn is blue

G_i = The ith ball drawn is green

E_i = The ith ball drawn is even

O_i = The ith ball drawn is odd

Find

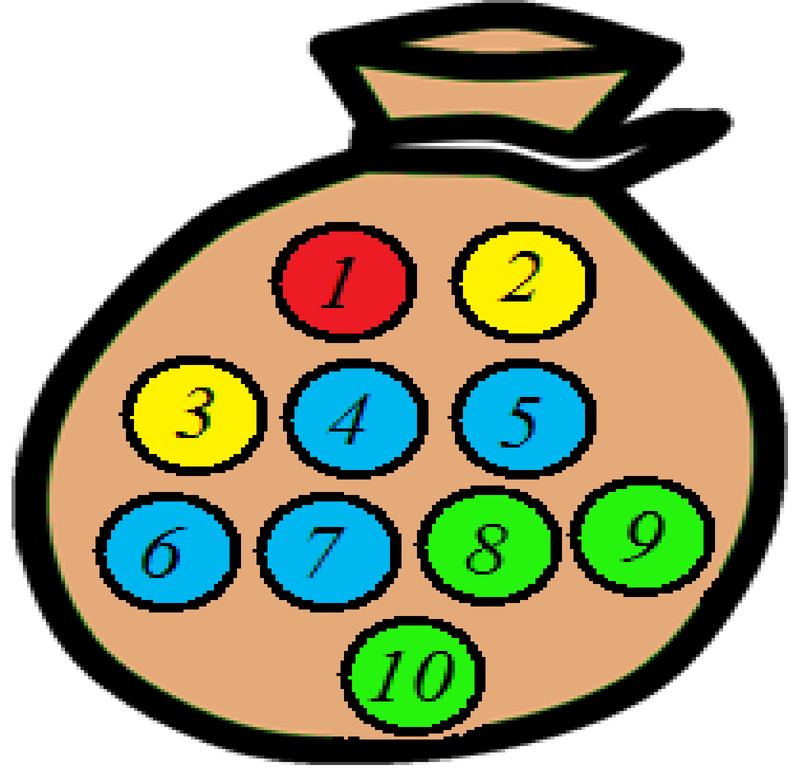
$P(R_2 \mid R_1), \;\; P(G_2 \mid B_1)$

Question: Are the events R_2 and B_1 independent?

Ex 6:

Experiment

Draw 2 balls from the bag one by one without replacement



Events

R_i = The ith ball drawn is red

Y_i = The ith ball drawn is yellow

B_i = The ith ball drawn is blue

G_i = The ith ball drawn is green

E_i = The ith ball drawn is even

O_i = The ith ball drawn is odd

Find

$$P(R_2 \mid R_1),\ \ P(G_2 \mid B_1),\ P(B_2 \mid B_1),\ P(R_2 \mid Y_1)$$

Ex 7:

Experiment

Draw a single card from a standard poker deck

Events

A = Draw a heart

B = Draw a black card

C = Draw a red face card

D = Draw a king

E = Draw a card that has a number on it that is less than 5

Find

$P(C|A)$, $P(B|D)$, $P(D|C)$

Question:

Are the events A and D independent?

Ex 8: A detective is trying to solve a murder case. At the outset of his investigation he knows that the murderer is one of the 200 people in the following table.

		City of Residence		
	Murder Suspects	Montebello	Pico Rivera	Total
Gender	Male	105	35	140
	Female	45	15	60
	Total	150	50	200

Questions:

a) What is the probability that at the outset of the investigation the murdered is female?

b) Suppose that after further narrowing down the suspects, the investigator has determined that the murderer lives in Pico Rivera. Now what is the probability that the murderer is female?

c) Are the events “murderer is female” and murderer lives in Pico Rivera independent?

Formulas for $P(A \cup B)$, $P(A \cap B)$, & $P(\bar{A})$

Formulas for $P(A \cap B)$ (the multiplication rule)

1) If A and B are independent (i.e. $P(B|A) = P(B)$), then

$$P(A \cap B) = P(A) \cdot P(B)$$

2) If A and B are NOT independent (i.e. $P(B|A) \neq P(B)$), then

$$P(A \cap B) = P(A) \cdot P(B|A)$$